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A Stochastic Model of Reliability for Bivariate Gamma Distributions with the Levels and Circadian Rhythmicity of Acth As A Function of Alcoholism Due To Human Stress

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Abstract
The theoretical study for the effect of stress on ACTH levels of individuals with a family history of alcoholism may present a dysfunction in the activity of the HPA axis is investigated. The stress strength model describes the life of a component which has a random strength Y and is subjected to a random stress X. R =P_r (X< Y) is a measure of the component reliability is used to find the levels of ACTH, Cortisol and β-endorphin.
Keywords: ACTH, CORTISOL, β ENDORPHIN, GAMMA DISTRIBUTIONS
Mathematical subject classification: $60G_{XX}$, $62H_{XX}$, $62P_{XX}$.

I. INTRODUCTION

The theoretical study for the effect of stress on HPAaxis dysfunction is associated with alterations in the pattern of the circadian (24h) secretions of Adrenocorticotropic hormone (ACTH) cortisol and β-endorphin. Men with High risk (HR) or without Lowrisk (LR) family history of alcoholism are participated in this study. Cherian's Bivariate gamma distributions, the stress strength model describe the life of a component which has a random strength Y and is subject to a random stress X. The component fails at the instant that the component will function satisfactorily whenever Y>X. $R = P_r(X < Y)$ is a measure of the component reliability and it used to observe the HR and LR participant of the plasma ACTH, cortisol and β-endorphin.

II. APPLICATION

Epidemiological studies clearly indicate the genetic factors and family history play a significant role in determining a person's vulnerability for high alcohol consumption and alcoholism. Twin studies, adoption, and cross-fostering studies as well as detailed pedigree analyses all suggest that alcoholism "runs" in families and that there are multiple genes which interact with environmental factors in a complex manner to increase or decrease an individual's vulnerability to develop alcoholism. Using families with either a strong positive or negative history of alcoholism, investigations were performed to determine whether certain genetically transmitted traits or markers are highly associated with the incidence of alcoholism. These markers could be either behavioral, such as impulsive or violent behavior, physiological, such as electroencephalogram abnormalities and body sway, or biochemical, such as enzymes, hormones, neurotransmitters, and nueromodulators.

Stress is any perceived challenge, either physical or psychological, real or imagined, that will disturb an individual's relatively steady internal environment, known as homeostasis, which allows optimal functioning of the organism.

One o the systems greatly affected by stress is the hypothalamic-pituitary-adrenal (HPA) axis. The HPA-axis response to stress is coordinated by complex interactions of various neurotransmitter systems designed to help an individual cope with a perceived threat and then return the organism to homeostasis. Although the mechanisms underlying the relationship between stress and alcohol consumption are not well understood, biological systems affected by both alcohol and stress most likely play an important role [2]. Both alcohol and stress affect the HPA-axis and the β-endorphin system. The major hormones of the HPA-axis are corticotrophin-releasing hormone (CRH), adrenal corticotropic hormone (ACTH), and cortisol. Proopiomelanocortin (POMC) is the common precursor for ACTH and β-endorphin [4].

Both stress and alcohol modulate the activity of these neurotransmitter systems leading to increased release of CRH and subsequently of β-endorphin, ACTH and Cortisol. Plasma ACTH,Cortisol and β-endorphin levels exhibit a circadian rhythm with low hormone levels in the late evening, reaching their minimum during the first several hours after sleep and their maximum plasma hormone levels at about 0800 hours in the morning.

Nonalcoholic participant with a family history of alcoholism [high risk (HR)] exhibited lower basal

plasma β-endorphin and ACTH levels than individuals without alcoholic relatives [low risk (LR)] [5].

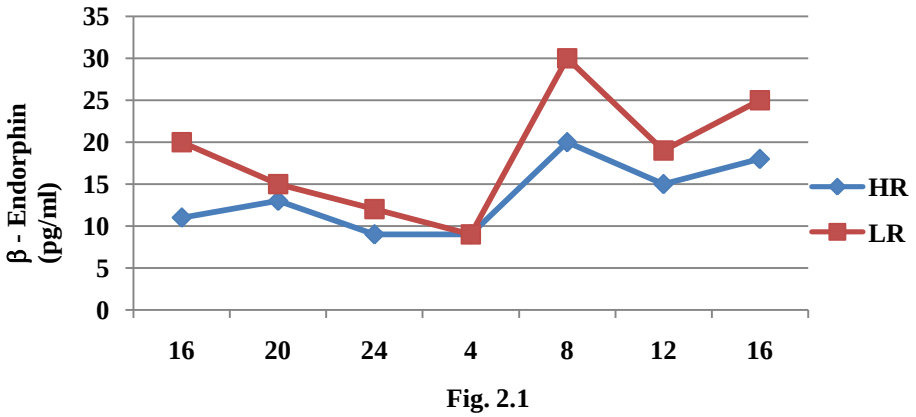

Fig. 2.1

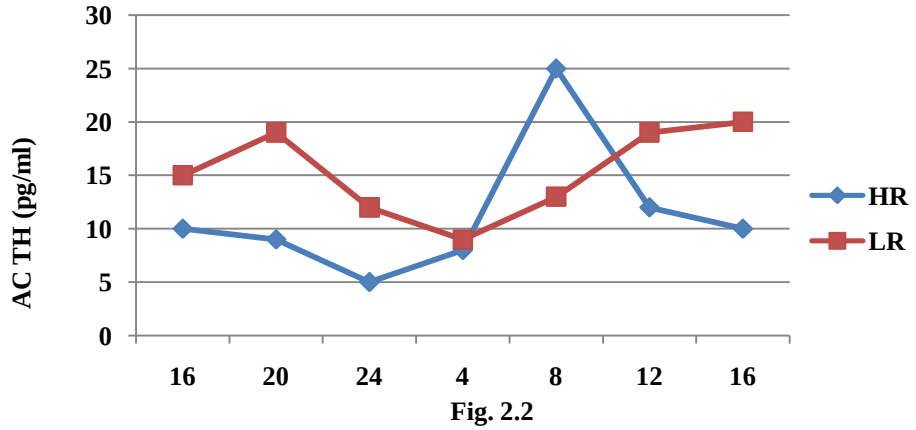

Fig. 2.2

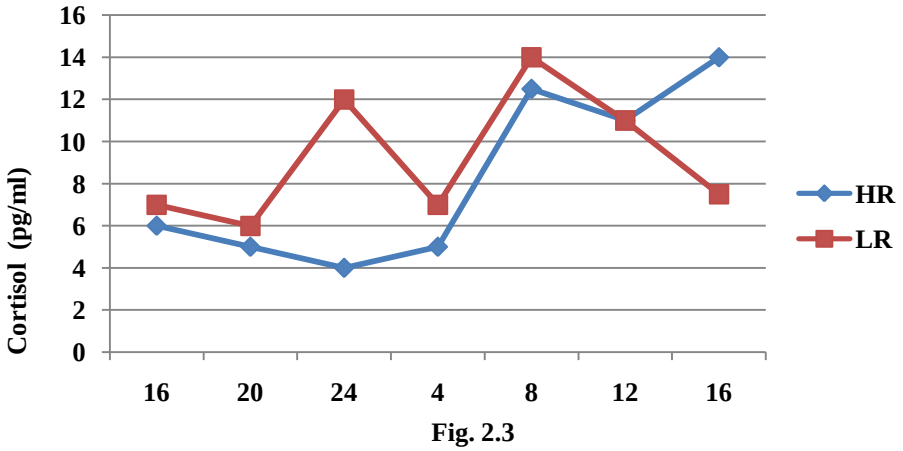

Fig. 2.3

Figure 2.1,2.2 & 2.3 illustrates the 24 hrs. secretion profile of β- endorphin, ACTH and Cortisol level of Low Risk (LR) and High Risk (HR) participants.

III. MATHEMATICAL MODEL

Bivariate gamma distribution arises tractable "lifetime" model in many areas, including life testing and telecommunications [1].

$R = P_r(X < Y)$ is a measure of the component reliability [6].

The calculation of R has been extensively investigated when X and Y are dependent random variables.

(X, Y) has a bivariate gamma distribution with joint probability density function (pdf) f and joint survivor function F .

$$R = \int_0^\infty \int_x^\infty f(x, y)dydx.$$

Calculation of R make use of a number of special functions. They are the complementary incomplete gamma function defined by

$$\Gamma(a, x) = \int_x^\infty t^{a-1} \exp(-t)dt,$$

The confluent hypegeometric function $({}_1F_1)$ defined by

$${}_1F_1(a; b; x) = \sum_{k=0}^{\infty} \frac{(a)_k x^k}{(b)_k k!}$$

The Guass hypergeometric function $({}_2F_1)$ defined by

$${}_2F_1(a; b; x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k x^k}{(c)_k k!}$$

The generalized hypergeometric function $({}_2F_2)$ defined by

$${}_2F_2(a; b; x) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k x^k}{(c)_k (d)_k k!}$$

The kummer function defined by

$$\Psi(a, b, x) = \frac{1}{\Gamma(a)} \int_0^\infty t^{a-1}(1+t)^{b-a-1} \exp(-zt)dt,$$

And the modified Bessel function of the first kind defined by

$$I_m(z) = \sum_{k=0}^{\infty} \frac{z^{2k+m}}{2^{2k+m} k!\Gamma(k+m+1)}$$

Where $(e)_k = e(e+1)....(e+k-1)$ denotes the ascending factorial.

Cherian's bivariate gamma distribution has the joint pdf specified by [3]

$$f(x, y) = K \exp(-x) \exp(-y) \int_0^{\min(x,y)} (x-z)^{\theta_1 - 1} (y-z)^{\theta_2 - 1} z^{\theta_3 - 1} \exp(z)dz \quad (1)$$

$$for\ \mathrm{x} > 0, \mathrm{y} > 0, \theta_1 > 0, and\ \theta_3 > 0, \text{ where K denotes the normalizing constant given by}$$

$$\frac{1}{K} = \Gamma(\theta_1)\Gamma(\theta_2)\Gamma(\theta_3) \quad (2)$$

For this distribution , the form of R can be expressed as

$$\begin{aligned} R &= K \int_0^\infty \int_x^\infty \int_0^{\min(x,y)} (x-z)^{\theta_1 - 1} (y-z)^{\theta_2^{-1}} z^{\theta_3 - 1} \exp(z-x-y)dzdydx \\ &= K \int_0^\infty \int_0^x (x-z)^{\theta_1 - 1} \exp(-x) z^{\theta_3 - 1} \exp(z) \left\{ \int_x^\infty (y-z)^{\theta_2 - 1} \exp(-y)dy \right\} dzdx \\ &= K \int_0^\infty \int_0^x (x-z)^{\theta_1 - 1} \exp(-x) z^{\theta_3 - 1} \exp(z) \left\{ \int_{x-z}^\infty w^{\theta_2 - 1} \exp(-w)dw \right\} dzdx \\ &= K \int_0^\infty \int_0^x (x-z)^{\theta_1 - 1} \exp(-x) z^{\theta_3 - 1} \Gamma(\theta_2, x-z)dzdx \\ &= K \int_0^\infty x^{\theta_1 + \theta_3 - 1} \exp(-x) \int_0^1 t^{\theta_1 - 1} (1-t)^{\theta_3 - 1} \Gamma(\theta_2, xt)dtdx \text{ [8]} \end{aligned} \quad (3)$$

Where the transformations are $\mathrm{w} = \mathrm{y} - \mathrm{z}$ and $\mathrm{t} = (\mathrm{x} - \mathrm{z})/\mathrm{x}$ have been applied.

Application of Lemma : $For\ \alpha > 0 \text{ and } \beta > 0,$

$$\int_0^a x^{\alpha - 1} (a-x)^{\beta - 1} \Gamma(\gamma, cx)dx$$

$$= a^{\alpha+\beta-1}\Gamma(\gamma)B(\alpha, \beta) - \gamma^{-1} a^{\alpha+\beta+\gamma-1} c^\gamma B(\beta, +\gamma)_2 F_2(\gamma, \alpha+\gamma; \gamma+1, \alpha+\beta+\gamma; -ac)$$

shows that the inner integral in (3) can be calculated as

$$\int_0^1 t^{\theta_1 - 1} (1-t)^{\theta_3 - 1} \Gamma(\theta_2, xt)dt$$

$$= \Gamma(\theta_2)B(\theta_1, \theta_3) + \theta_2^{-1} B(\theta_3, \theta_1 + \theta_2) x^{\theta_2} {}_2F_2(\theta_2, \theta_1 + \theta_2; \theta_2 1; \theta_1 + \theta_2 + \theta_3; -x) \quad (4)$$

and thus R can be defined as [7]

$$\mathrm{R} = \frac{1}{\Gamma(\theta_1)\Gamma(\theta_3)} + \theta_2^{-1} B(\theta_3, \theta_1 + \theta_2) I, \tag{5}$$

Where I denotes the integral

$$I = \int_0^\infty x^{\theta_1+\theta_2+\theta_3-1} \exp(-x)\, {}_2F_2(\theta_2, \theta_1+\theta_2; \theta_2+1; \theta_1+\theta_2+\theta_3; -x)dx. \tag{6}$$

This integral can be calculated by an application of Lemma For $c > 0$ and $\sigma > w$,

$\int_0^\infty x^{c-1} \exp(-\sigma x)\, {}_2F_2(a, b, d, c, wx)dx = \sigma^{-c}\Gamma(c)\, {}_2F_1\left(a, b, d; \frac{w}{\sigma}\right)$ to get

$$I = \Gamma(\theta_1+\theta_2+\theta_3)\, {}_2F_1(\theta_2, \theta_1+\theta_2; \theta_2+1; -1) \tag{7}$$

and hence it follows from (5) that the form of R for Cherian's bivariate gamma distribution is given by

$$R = 1 + \theta_2^{-1}\Gamma(\theta_1+\theta_2)\Gamma(\theta_3)\, {}_2F_1(\theta_2, \theta_1+\theta_2; \theta_2+1; -1)$$

IV. RESULT

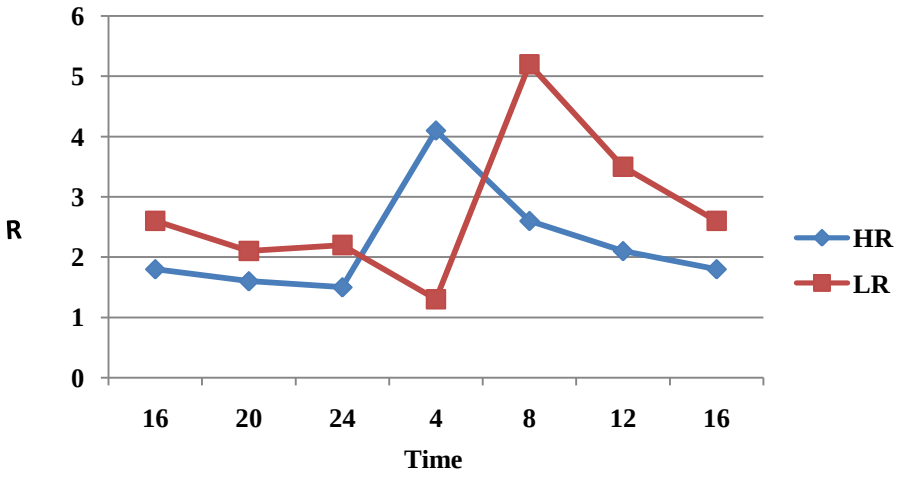

Fig. 4.1

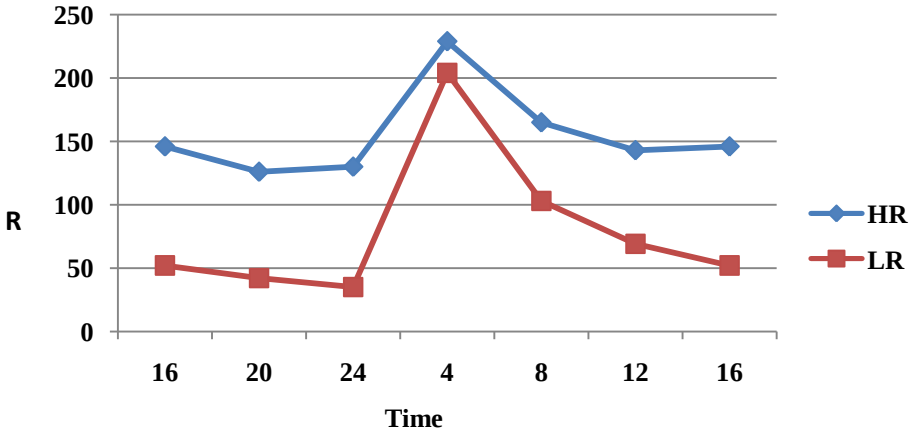

Fig. 4.2

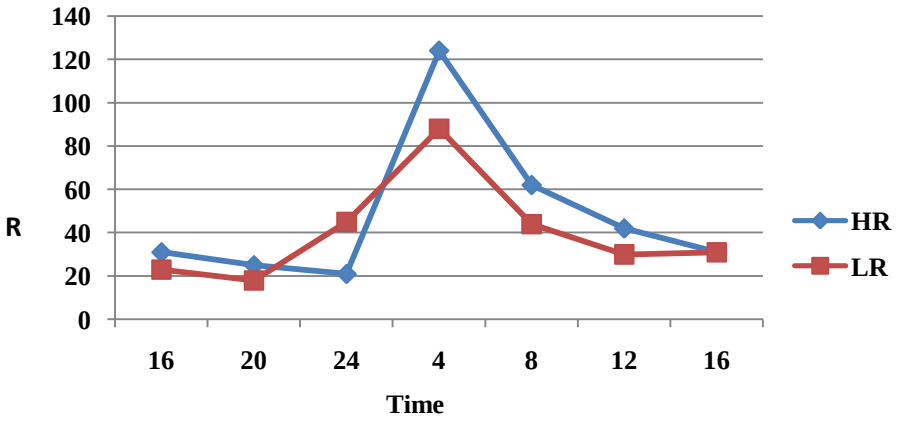

Fig. 4.3

The 24 hrs. secretion profile of β-endorphin , ACTH and Cortisol of LR and HR in the medical data are to calculate

$$R = 1 + \theta_2^{-1}\Gamma(\theta_1 + \theta_2)\Gamma(\theta_3)_2 F_1(\theta_2, \theta_1 + \theta_2; \theta_2 + 1; -1)$$

where $\theta_1\ \theta_2\ \theta_3$ are the parameters is correlated with the gamma distribution and it is developed in the graph is fitted with real life data to obtain the level of ACTH, Cortisol and β-endorphin of LR and HR.

V. CONCLUSION

The HR and LR participants presented similar Circadian 24 hrs. Secretion patterns for ACTH, Cortisol & β- endorphin. Due to this pattern, compared to LR, HR participants presented lower concentration of plasma ACTH and of β- endorphin but not of Cortisol. Using bivariate gamma distribution relating the successive time intervals and sample values of ACTH, β-endorphin and Cortisol are used to estimate R by the parameters. LR participants presented lower concentrations of Cortisol and ACTH but not β-endorphin. The model is developed to find out the levels and hormone secretion and the results have been defined with a mathematical model fitted with the medical report.

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